Lesson Title	A Penny for your Shoe
Length of Lesson	50 minutes
Created By	Emily Burtnett
Subject	Geometry
Grade Level	10^{th}, 11^{th}, 12^{th}, High School Topics
State Standards	Geometry 4a
DOK Level	2
DOK Application	Solve real-world problems involving formulas for perimeter, area, distance, and rate.
National Standards	Use visualization, spatial reasoning, and geometric modeling to solve problems.
Graduate Research Element	Calculating the area of the droplet on the surface after impact.

Student Learning Goal:
They will calculate the area of a penny and approximate the area of their shoe by discovering how many pennies fit inside an irregular object (the bottom of their shoe). Students will learn to use measuring tools such as a ruler and a compass to approximate the area of an irregular object using the Area Addition Postulate. They will divide the irregular shape into regular shapes that they can recognize such as triangles, rectangles, circles, squares, trapezoids, and so forth. This demonstrates to the students that the Area Addition Postulate is a very accurate method. Students will also become familiar with measuring tools and using units. The ruler is used to not only draw lines, but to measure lengths, as well. Students will calculate the error between the two calculated areas to gain a better understanding of why and how errors are used and why they are necessary in engineering/mathematical applications.

Materials Needed (supplies, hand-outs, resources):
Rulers, compasses, calculators, paper (multiple sizes for larger shoes), extra shoes, pencils, 500 pennies

Lesson Performance Task/Assessment:
Students will trace the perimeter of their shoe onto a piece of paper. Using a compass and a ruler, they will divide the irregular object into recognizable shapes such as triangles, rectangles, circles, squares, trapezoids, and so forth in a fashion they see fit, working from top to bottom.

Student will measure and calculate the area of the circular side of a penny. They will then fill the outline of their shoe with as many pennies as they can fit inside without going over the perimeter. The area is calculated by counting the number of pennies inside the shoe and multiplying it by the area of the penny. This value will be recorded.

Using the ruler, the students will measure the lengths required to calculate the areas of each individual regular shape inside their shoe. The calculations will be written out and recorded on a separate piece of paper. When all the areas are calculated, the students will add them up to determine the approximated total area of their shoe; this is the Area Addition Postulate. The calculated value will be recorded. The two determined values of the area of the shoe will be compared using various techniques, but it will be obvious that the Area Addition Postulate gives a much more accurate approximation since the pennies leave a lot of empty space.

Lesson Relevance to Performance Task and Students:
Students will discover that geometry is *everywhere*, even in irregular shapes. They will also become more comfortable with measuring and using units. Calculating the error will demonstrate to them that in many real-life applications, quantities are approximated. By calculating the error, they can determine how accurate their approximation is.

Anticipatory Set/Capture Interest:
The instructor will hold up a shoe (taking off and holding up their own shoe would add a little humor to the scenario) and ask the students:

- Is it possible to calculate the area of the bottom of this shoe?
    - Is it easy?
        - If students say think it will be easy, ask "How accurate do you think it will be?"
        - If they think it will be difficult, continue with lesson and introduce the Area Addition Postulate.

Guided Practice:
The instructor will review the areas of regular shapes and go through the instructions of the activity. The instructor will also walk around the classroom and monitor students and assist as needed during the independent practice.

Independent Practice:
Students will calculate the area of their own shoes independently, but will be allowed to collaborate with their neighbors.

Remediation and/or Enrichment:
For remediation, the equations of the area of regular objects will be displayed for the class or given in a handout. If students are struggling to divide the irregular shape into regular shapes, the instructor will have a shoe already divided into regular shapes as a handout and students will then measure and calculate the areas. Or, this could be done to shorten the activity. Individual IEPs will be supported.

For enrichment, the instructor can point out how some "regular" shapes need to be assumed, especially where the shoe curves toward on the inside of the foot. For example,

it may be possible to generate a triangle where the hypotenuse is slightly curved rather than a straight line. The instructor can explain how the area of an n-sided polygon when n is very large can be approximated by the area of a circle. This can be translated into calculus, the limit as n goes to infinity of an n-sided polygon approaches pi times the square of the radius. Additionally, the instructor can demonstrate how evaluating the area under the curve of a function using integration is a more complex application of the Area Addition Postulate in calculus. Further enrichment would be to ask students to write the area of their shoe as a function of circles (the pennies) using the equation of a circle and approximating the area between the pennies to get a more exact measurement.

Check(s) for Understanding:
Discussion, question and answer. Have students share their results and error calculations. If some students' errors are significantly large, discuss why this might be and what may have gone wrong. Students will demonstrate the ability to use a compass to draw circles, a rule to draw lines/shapes and to measure lengths.

- Does your answer make sense to you? Why or why not?
- Why do you think the approximation with the pennies is such much different or less accurate?
- What other irregular shapes can you think of that we can use this method to approximate the area of? *(Trick question – fundamentally, this can be done with any irregular object, some irregular shapes will obviously be easier than others.)*
- Was there any *right* or *correct* way of dividing the shoe up into shapes?
- Does your answer make sense to you? What about your error?

Closure:

- Was calculating the area of your shoe easier than you expected?
- Can you think of any real-world applications where you might need to know the area of an irregular shape?

Just for fun: Give prizes to the students with the largest and smallest foot.

Possible Alternate Subject Integrations:

Teacher Notes:
http://www.rio.k12.wi.us/MATH/geo1.html
http://www.basic-mathematics.com/area-of-irregular-shapes.html