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| **Lesson Title** | Fundamentals of Fractals |
| **Length of Lesson** | 1 50-minute lesson |
| **Created By** | Sean Owens |
| **Subject** | Mathematics, Geometry |
| **Grade Level** | $9^{th}$ – $12^{th}$ |
| **State Standards** | $9^{th}$ - $12^{th}$ Pre-Algebra: 3d, 4a, 4b<br>Transition to Algebra: 3b<br>Algebra 1: 4a |
| **DOK Level** | DOK 2 – Transition to Algebra, Algebra 1<br>DOK 3 – Pre-Algebra |
| **DOK Application** | 2 – Identify Patters, Construct, Predict, Compare<br>3 – Construct, Explain Phenomena in Terms of Concepts, Hypothesize |
| **National Standards** | 9-12: B: Geometry<br>C: Measurement<br>D: Data Analysis and Probability |
| **Graduate Research Element** | Fractals exhibit a recursive behavior (smaller and smaller pieces) similar to that of the program I am designing for my research (smaller and smaller function blocks). |

**Student Learning Goal:**
After this lesson, students will be able to use recursion techniques to define fractal patterns.

This lesson addresses Mississippi 9-12 Mathematics standards: Pre-Algebra 3d; Transition to Algebra 3b; Algebra 1 4a; and Geometry 1a. It also addresses National 9-12 Mathematics standards B, C, and E.

State Standards: $9^{th}$ – $12^{th}$ Mathematics

Pre-Algebra – 3d: Solve real-world and non-routine problems involving congruent and similar figures.

Pre-Algebra – 4a: Solve real-world application problems that include length, area, perimeter, and circumference using standard measurements.

Pre-Algebra – 4b: Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Transition to Algebra – 3b: Apply proportional reasoning to determine similar figures and find unknown measures.

Algebra 1 – 4a: Solve real-world problems involving formulas for perimeter, area, distance, and rate.

National Standards: 9th – 12th Mathematics

B (Geometry):

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Apply transformations and use symmetry to analyze mathematical situations
- Use visualization, spatial reasoning, and geometric modeling to solve problems

C (Measurement):

- Apply appropriate techniques, tools, and formulas to determine measurements

D (Data Analysis and Probability):

- Develop and evaluate inferences and predictions that are based on data

**Materials Needed (supplies, hand-outs, resources):**

- Paper
- Ruler

**Lesson Performance Task/Assessment:**

Students will construct a fractal and calculate the geometric ratio defined by it. Then they will use the ratio to predict a future recursion value.

**Lesson Relevance to Performance Task and Students:**

This lesson will have students construct fractals demonstrating their understanding of recursion and similar polygons. The students will collect side length data, define the side length reduction ratio, and predict the side length at a theoretical level of recursion to demonstrate an understanding of ratios and geometric series prediction.

**Anticipatory Set/Capture Interest:**

As an anticipatory set, the instructor will show the students a series of images that demonstrate fractals in real world scenarios (conch shells, snowflakes, fern leaves) and ask the students to explain what they have in common. After discussion, the instructor can lead into an explanation of fractals and the guided practice section of the lesson.

**Guided Practice:**

In the guided practice section of the lesson, the instructor will explain the principals of fractals and how they are constructed. Then, he or she will demonstrate the fractal to be used by the students in the independent practice section (Koch fractal; see teachers notes) and explain the activity.

**Independent Practice:**

In the independent practice, students will construct fractals in teams to a set number of iterations (5 or 6). Afterwards, the students will measure and record the side length of the triangles at each iteration. From this data, the students will calculate the geometric ratio between each level of iteration and then predict the side length of a triangle at the 10th level of recursion.

**Remediation and/or Enrichment:**
Remediation: Individual IEP.
Enrichment: Have the students complete the activity with a different fractal.

**Check(s) for Understanding:**
What is a fractal? What is recursion? What is the geometric ratio defined by the fractal?

**Closure:**
In the closure portion of the lesson, the instructor will have the student teams compare their methods and results. Then, the instructor will discuss the prevalence of recursive structures in fields such as computer science and engineering.

**Possible Alternate Subject Integrations:**
None

**Teacher Notes:**

- Koch Fractal Explanation: http://en.wikipedia.org/wiki/Koch_snowflake
- In order to reduce the need for erasing, students can be instructed to use colored pencils/markers to draw the fractal using a different color at each level of recursion.
- Other fractals that can be used: Sierpinski triangle, Heighway dragon